QUARTERLY PROGRESS REPORT NO. 1

OPTIMUM CONSTRUCTION OF REINFORCED PLASTIC CYLINDERS SUBJECTED TO HIGH EXTERNAL PRESSURE

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FOREWORD

This is the first quarterly progress report on the work performed by Douglas Aircraft Company under BuShips Contract NObs-88425 during the period from 6 March 1963 to 6 June 1963. This contract is a 12-month research and development program under the supervision of the Structural Mechanics Laboratory, Code 733, David Taylor Model Basin with Dr. T. Reynolds acting as Technical Director.

The program is being conducted by the Solid Mechanics Branch, Missile Research Department, Advanced Missile Technology Division of Douglas Aircraft Company. Major responsibility for the program resides with H. R. Jacobson, Study Director, assisted by F. M. Tokiro.

ABSTRACT

The purpose of this program is to develop and prove out a theoretical method for accurately predicting the critical buckling strength of externally pressurized cylinders made from orthotropic materials and to find the optimum construction of filament reinforced plastic (FRP) cylinders under such loading.

A description of the program and its significance to the overall BuShips Deep Submergence program and a summary of the work accomplished to date and that to be performed in the next period is given. Theoretical analytical expressions for predicting elastic buckling collapse and elastic constants of FRP cylinders are presented and discussed. Results of initial buckling and discontinuity stress analysis of proposed test cylinders are presented.

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NOMENCLATURE

B_x, B_y	=	extensional stiffness of orthotropic cylinder in longitudinal and circumferential directions, pounds per inch
b	=	circumference of cylinder, inches
D_x, D_y	=	flexural stiffness of orthotropic cylinder in longitudinal and circumferential directions, inch-pounds
D_{xy}	=	twisting stiffness or orthotropic plate in xy-plane, inch-pounds
E	=	Young's modulus, pounds per square inch
E_x, E_y	=	elastic modulus of orthotropic cylinder in longitudinal and circumferential directions, pounds per square inch
G	=	shear modulus of isotropic plate, pounds per square inch
G_{xy}	=	shear stiffness of orthotropic plate in xy-plane, pounds per inch
G'_{xy}	=	shear modulus of orthotropic plate in xy-plane, pounds per square inch
I_x	=	distributed moment of inertia in longitudinal direction, inches cubed
I_y	=	distributed moment of inertia in circumferential direction, inches cubed
K_y	=	buckling coefficient for cylinders under external pressure
L	=	length of cylinder, inches
L_D, L_E	=	mathematical operators
m	=	number of half-waves in longitudinal directions
n	=	number of half-waves in circumferential directions
N_x	=	load in longitudinal direction, pounds per inch
N_y	=	load in circumferential direction, pound per inch
p_{cr}	=	critical buckling pressure, pounds per square inch
r	=	radius of curvature of median surface of cylinder, inches
t	=	monocoque cylinder thickness, inches

w	=	radial displacement, inches
x, y	=	longitudinal and circumferential coordinates, inches
Z_y	=	curvature parameter for orthotropic cylinder under external pressure $\left(= \frac{L^2}{r}\sqrt{\left(1 - \mu_x\ \mu_y\right)}\sqrt{\frac{B_x}{D_y}}\right)$
λ	=	ratio of volume of glass to total volume
λ'	=	half-wavelength of buckle in longitudinal direction $(= \frac{L}{m})$, inches
μ_x, μ_y	=	Poisson's ratios for orthotropic cylinder associated with flexure
μ'_x, μ'_y	=	Poisson's ratios for orthotropic cylinder associated with normal strains

SUBSCRIPTS

g	=	refers to glass
H	=	hoop wrap
HW	=	helical wrap
L	=	longitudinal direction of laminate
LT	=	plane of the laminate
r	=	refers to resin
T	=	transverse direction of laminate
x, y, z	=	longitudinal, circumferential, and radian directions
xy	=	plane of cylinder wall

1.0 INTRODUCTION AND SUMMARY

1.1 Background

As the desired depth of descent for deep submergence structures increases the structural weight required, using currently available materials, greatly increases and the ability to carry a useful payload diminishes. The Bu Ships program on Filament Reinforced Plastics (FRP) for Deep Submergence Structure is aimed at developing this promising lightweight structural material for long-time compression loading and assessing the feasibility of its use in deep submergence applications. At the present time the results of these programs indicate that the short time compressive strength-weight ratio of FRP laminate is superior to those for available steel and titanium alloys while retaining the advantage of simplicity of fabrication, especially for thick wall constructions.

As the compressive strength of FRP laminates has risen during the course of the Bu Ships programs it has pointed up one of the more important design aspects of externally pressurized structure in general and those fabricated from FRP in particular; namely, the problem of elastic instability failure or buckling collapse of shell structures. When this type of failure occurs at pressures lower than those required to produce compressive stress failure, the structure required is heavier than necessary. Theoretically, for most efficient utilization of materials, a structure should be designed to fail by buckling and compressive stress simultaneously, rather than by either one.

In actual practice, underwater structures such as submarine hulls are designed to resist buckling loads higher than those needed to cause compression failure.

In order to design efficient deep submergence structure from FRP laminates, a method is needed for accurately predicting collapse pressure. In addition, if minimum weight structures are required, a method is needed which will optimize the structural parameters of the design with respect to weight. Development of such methods for FRP is complicated by the orthotropic and in some cases non-homogeneous nature of the laminates. Because of the complexity of developing methods of analysis for orthotropic materials, methods developed for isotropic materials are currently extensively used to predict buckling collapse. Use of such methods generally result in considerable error in design and does not permit the selection of the most efficient structural arrangement.

Development of theoretical analytical methods for FRP structures has been underway at the Douglas company for some time. Independent research and development programs have developed preliminary analytical treatments for the prediction of the collapse pressure for monocoque FRP cylinders under external pressure.

1.2 Program Plan

The purpose of this program is to develop and prove out a theoretical method for accurately predicting the buckling strength of externally pressurized orthotropic cylinders such as those constructed of FRP. In addition, the optimum laminate construction or layup pattern for buckling resistance will be determined and weight efficiencies of various types of laminate constructions will be compared.

The program can logically be divided into three (3) main phases

1. Theoretical and Design Phase

   The preliminary theoretical methods developed by Douglas will be used as a basis for the development of an accurate buckling prediction method. The existing analysis will be revised and/or extended as indicated by additional studies and by test results. As a part of this phase the detail design and analysis of small (6" ID x 12" long) cylinders to be tested in Phase 2 will be performed. Figure 1 and Table 1 from reference [1] show details of the types of constructions to be used as well as the predicted strength of possible configurations of these laminate types. Shown on the curves are points which have been selected for test substantiation of theoretical buckling pressures. All but two of the test cylinders will be designed to ensure buckling failure. The two highest buckling strength constructions will be incorporated into thick wall cylinders designed to determine their compressive strength.

2. Fabrication and Test Phase

   Two cylinders each of the eleven different configurations (A-L) shown in Figure 1 will be fabricated and tested to collapse under external hydrostatic pressure. The elastic constants of the cylinders and the relationships between then will be determined during testing as well as collapse pressures and compared with the theoretically determined values. After all configurations have been tested and the analysis methods proved out, one specimen each of the optimum construction and a high buckling strength alternate construction (preliminarily shown as B and E) will be fabricated and tested to buckling failure and one thick wall specimen each of these constructions (preliminarily shown as M and N) will be fabricated and tested to determine ultimate compressive strengths. A total of twenty-six (26) cylinders will be fabricated and tested.

   Figure 2 shows the newly revised program schedule. The fabrication and testing is devided into four sub-phases with the following breakdown;

   1. Group I Cylinders - Configurations A, B, C and D - 2 each
   2. Group II Cylinders - Configurations E, F, G and H - 2 each
   3. Group III Cylinders - Configurations J, K and L - 2 each
   4. Group IV Cylinders - Optimum construction and high buckling strength alternate with compressive strength test specimens of each-tentatively chosen as configurations B, E, M and N - 1 each

   Materials to be used will be identical to those used in other Bu Ships programs with the exception that Douglas collimated pre-preg tapes will be fabricated and used in this program instead of the commercially available pre-preg tape currently being used by other contractors.

3. <u>Data Reduction and Report Phase</u>

This is a continuing effort carried on concurrently with the main efforts of the program and will include monthly, quarterly, and final reports, as well as a special report outlining the theory, test data and comparison with theory of the work performed in Phase 1 and 2 above. All test data and comparison with theoretical predictions will also be reported in regular reports as soon as it becomes available.

1.3 Summary of Work Accomplished

Although this contract is to run from 6 March 1963 to 6 March 1964, a formal contract was not received by the Douglas Company until 23 April 1963. However, authority to proceed was received by the Study Director on 17 April 1963 and work was started immediately thereafter. The work accomplished during this quarter was therefore seriously curtailed. A new schedule was developed which will permit completion of the program within the original contract period and will permit completion of all fabrication and testing work by January 1, 1964.

The work accomplished during this period includes:

1. Administrative start-up and reschedule of the program.
2. Design of fabrication tooling, release of tool drawings and ordering of tooling materials. Fabrication of the mandrels for test cylinders has been started and work is ahead of schedule.
3. Theoretical studies on the buckling prediction method and elastic constant equations have been started. Preliminary analysis of the proposed test cylinder constructions has also been performed and some results are included in this report.
4. Analysis of the discontinuity stresses expected in the ends of the test cylinders due to test fixture restraint has been started. Results for some of the test cylinders are included in this report.
5. Single end E-HTS glass rovings for fabrication into Douglas tape of twenty (20) end E-HTS glass rovings for wet winding of helically wound test cylinders have been ordered. Fabrication of Douglas collimated tape has been started.

2.0 THEORETICAL STUDIES

2.1 Buckling Theory

The equations which are the theoretical basis for the proposed buckling prediction method were derived prior to this program under Douglas Independent Research and Development programs [2, 3, 4]. However, some changes in the expressions for elastic constants as a result of testing prior to and analytical work performed under this contract have resulted in improved expressions for some of the elastic constants required for use in the buckling equation.

2.1.1 General

The analytical determination of the critical buckling pressure of FRP cylinders is based on two analytical tools. The first is a mathematical representation of the deflection of the cylinder under the loads placed on it. The deflection of the cylinder is controlled by the elastic properties (due both to material and shape) of the shell. The second tool is a set of expressions which describe the elastic properties of FRP laminates. Due to the orthotropic and nonhomogenous nature of FRP laminates, expressions describing the elastic properties are complex and can be different for each direction and type of construction. For this reason, approximations and expressions based on isotropic materials have been extensively used in the past. However, as will be shown in the following discussion, critical buckling loads of orthotropic cylinders are very sensitive to variation or error in the elastic constants and stiffness of the laminate.

In FRP laminates, both the modulus and moment of inertia of the construction can vary fairly widely. The modulus of any particular layer is a function of resin content and fiber and resin elastic properties. The moment of inertia or "configurational" stiffness is a function of thickness and arrangement of all the individual layers within the wall thickness. This latter affect means that flexural stiffness and therefore buckling strength of the cylinder can be different even for two hoop to axial layup patterns which have the same proportions and which therefore should have identical extensional stiffness and compressive strength.

2.1.2 Derivation of Buckling Equation

Based on a small deflection theory for buckling of orthotropic cylindrical shells by Stein and Myers [5] the equilibrium condition for a cylinder under arbitrary loading is given by the following differential equation [2].

$$L_D w + \frac{G_{xy}}{r^2} L_E^{-1} \frac{\partial^4 w}{\partial x^4} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial x^2} = 0 \qquad (1)$$

where w = radial displacement of cylinder
r = mean radius of cylinder

L_D and L_E are linear differential operators defined by

$$L_D = \frac{D_x}{1-\mu_x \mu_y} \frac{\partial^4}{\partial_x^4} + \left(\frac{\mu_y D_x}{1-\mu_x \mu_y} + 2 D_{xy} + \frac{\mu_x D_y}{1-\mu_x \mu_y} \right) \frac{\partial^4}{\partial x^2 \partial y^2}$$

$$+ \frac{D_y}{1-\mu_x \mu_y} \frac{\partial^4}{\partial_y^4}$$

and

$$L_E = \frac{G_{xy}}{B_y} \frac{\partial^4}{\partial_x^4} + \left(1-\mu'_x \frac{G_{xy}}{B_x} - \mu'_y \frac{G_{xy}}{B_y} \right) \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{G_{xy}}{B_x} \frac{\partial^4}{\partial_y^4}$$

and L_E^{-1} is the inverse operator defined by

$$L_E^{-1} (L_E w) = L_E (L_E^{-1} w) = w$$

N_x = axial load in pounds per inch of circumference

$= \frac{pr}{2}$

N_y = circumferential load in pounds per inch of axial length

$= pr$

The B's, D's, G's and u's are elastic constants of the cylinder defined under nomenclature.

The cylindrical shell under external hydrostatic pressure or external radial pressure exhibits four ranges of buckling behavior. 1) The lower limit corresponds to the short cylinder region. The behavior in this region is similar to flat plate buckling. This type of behavior is typical of isotropic cylinders of short length or orthotropic cylinders with a very high circumferential bending stiffness. 2) The transition range covers the region between the long and short cylinder range and represents an interaction of the two buckle modes. 3) The long cylinder range represents cylinders of moderate length and/or moderate circumferential bending stiffness. The buckle mode consists of one half wave in the longitudinal direction and many waves around the circumference of the cylinder. 4) The upper bound of buckling behavior is represented by the very long cylinder which buckles into two circumferential waves similar to a ring. For purposes of this program, only the long cylinder range will be considered.

Equation (1) can be solved for the case of a long cylinder under external hydrostatic pressure (Figure 3) by setting $N_x = N_y/2$ and assuming a displacement function $w = w_o \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$.

The general solution is shown in Appendix A for both radial and hydrostatic pressures. Results for both types of loading result in the identical expression for the critical buckling pressure;

$$P_{cr} = \frac{5.51}{R^{3/2}L} \left(\frac{D_y}{1-\mu_x \mu_y} \right)^{3/4} \left(B_x \right)^{1/4} \tag{2}$$

where

$$Z_y = \frac{L^2}{r}\sqrt{\frac{B_x \, (1-\mu_x \mu_y)}{D_y}} > 100$$

is a curvative parameter which defines the buckling range for long cylinders and includes affects of both curvative and stiffness on the buckling mode of the cylinder and,

B_x = extensional stiffness of the cylinder in the axial direction

$$= \sum_{i=1}^{i=n} E_{x_i} t_i$$

D_y = bending or flexural stiffness in the circumferential direction

$$= \sum_{i=1}^{i=n} E_{y_i} I_i$$

μ_x = Poisson's Ratio of the cylinder wall in the axial direction due to flexure

$$= \frac{1}{D_y} \sum_{i=1}^{i=n} E_{y_i} I_i \mu_{x_i}$$

μ_y = Poissons Ratio of the cylinder wall in the circumferential direction due to flexure

$$= \frac{1}{D_x} \sum_{i=1}^{i=n} E_{x_i} I_i \mu_{y_i}$$

E_{x_i}, E_{y_i}, μ_{x_i}, and μ_{y_i} are the extensional elastic moduli and Poisson's ratios of the ith layer and are given in Appendix B. Layers i = 1 through i = n can be composed of circumferential or hoop wraps (H), axial or longitudinal wraps (L), or helical wraps (HW) of fiber and resin composite layers.

The assumptions made in deriving equation (2) are that the shell is thin walled so that terms containing $(\frac{t}{r})^2$ can be neglected, that there are many waves in the hoop direction so that the approximation $(n^2-1) \approx n^2$ can be made where n is the number of half waves in the circumferential buckle pattern, that the ratio of axial to circumferential wave length is large compared to unity in order that only higher order terms involving the buckle aspect ratio need be retained.

The significance of this equation is that it shows there are two different types of stiffness which affect the critical buckling pressure of the cylinder (axial extensional stiffness, B_x, and hoop bending stiffness, D_y) and they affect the buckling strength to differing degrees. Therefore two cylinders of the same dimensions, weight and ratio of hoop to axial layers can have different buckling strengths if the placement or arrangement of these layers is such that different hoop bending stiffnesses result. Furthermore, examination of the equations for the elastic constants of the laminates (Appendix B) show that different proportions of glass and resin and differing elastic constants for the basic ingredients of glass fiber and resin will also affect the buckling strength of otherwise identical cylinders. Hence, the importance of knowing within close limits the elastic properties and proportions of the materials going into the fabrication of FRP laminates.

Using equation (2), a number of different types of FRP laminate arrangements for equal weight cylindrical constructions have been checked for comparative buckling strength in Reference [3]. Figure 4 shows the types of constructions analyzed which have been selected for inclusion in this program. Figure 1 shows the relative buckling strengths obtained for varying proportions of hoop to axial laminates for these constructions. In this figure, all buckling pressures are compared to the construction showing the highest strength. The analysis was performed for a given glass to resin volume ratio (2:1 or λ = 67% glass by volume) and a given ratio of resin to glass elastic moduli $(\frac{Er}{Eg} = 0.05)$. The resulting curves clearly indicate the relative efficiency of each type of construction in resisting buckling failure and also indicates the particular configuration of each type which results in the highest buckling strength.

It should be stressed here that this analysis involves only the <u>elastic buckling strength</u> of the cylinder and makes no assessment whatever concerning the materials' ultimate compressive strength.

2.2 Elastic Constants

As indicated in the preceeding sections, accurate expressions are required for the elastic constants of the laminate if accurate predictions of the buckling pressure of the cylinder are to be made. The preliminary work preceeding this program includes the derivation of mathematical representations of the elastic constants of individual layers of FRP [4] and multiple layer constructions. Appendix B shows the results of this work. However, these expressions are based on assumptions and simplified models in order to simplify the derivations. The results of Douglas test programs aimed at substantiating the accuracy of these expressions has indicated significant differences between theoretical and test results for some of these expressions. The initial theoretical studies performed under this contract include a review of the assumptions made and the models used in the derivation of the expressions for the moduli of elasticity and Poisson's ratios of unidirectional laminates. The studies have thus far resulted in improved expressions for the transverse compression and shear modulus, E_T and G_T, based on an improved model representing the FRP laminate. Figure 5 shows the original and improved models and the resulting expressions for longitudinal and transverse moduli.

The models used in Figure 5 are simplified in order to simplify the derivation, using in-line square fibers (of equivalent area) in place of the actual conditions of round fibers and something approaching a hexagonal packing arrangement. If significant unaccounted for discrepancies continue to be found between theoretical and test values of the elastic constants after these revisions, it may be necessary to revise the model again to bring it into closer agreement with actual conditions.

3.0 TEST CYLINDER DESIGN

3.1 Buckling Considerations

Since the primary purpose of this program is to develop a buckling prediction method, all test cylinders of the various constructions are being designed to insure buckling failure except for the cylinders which will check the compressive strengths of the two high buckling strength constructions mentioned previously. All the cylinder configurations indicated by letters A through L in Figure 1 have been selected for test to buckling collapse under external hydrostatic pressure. In order to obtain failures without excessive influence from thick wall stresses an $\frac{r}{t}$ ratio of 20 has been selected. In order

to insure that all cylinders are in the "long cylinder" range as defined by the Z_y or curvature parameter, a test length of at least 10 inches has been chosen. Table 2 shows the results of the buckling analysis on the test configurations. Values of Z_y, theoretical buckling pressure, relative buckling strength, and average hoop stress of each of the configurations to be tested are included. These values are based on the following dimensions and physical properties for all the cylinders and their construction:

I.D. = 6.000 in.

t = 0.150 in.

l = 10.5 in. (unsupported length)

λ = 67% glass by volume

E_r = 500,000 psi μ_r = 0.36

E_g = 10.0×10^6 psi μ_g = 0.2

3.2 Discontinuity Stresses

The external pressure test fixture which will be used in this program makes use of steel end fittings which perform the functions of loading the cylinder axially, providing the seal against external pressure and maintaining the circularity of the ends of the cylinder.

The last function is of special interest in the design of the test cylinders since the "plug" ends of the fixtures perform this task by preventing all radial deflection, uniform and non-uniform. This restraint induces local discontinuity stresses in the ends of the cylinder which could cause strength failure by exceeding either the axial compressive strength or the shear strength of the cylinder wall in the axial direction. The latter type of failure has been reported for thick wall cylinders tested by some of the contractors involved in the various BuShips FRB program and stems from the relatively low shear strength of FRP laminates.

In order to assure test cylinder designs which will fail by buckling only, cylinders are being designed to avoid high discontinuity stresses at the test fixture. Since the buckling test cylinders are relatively thin and the wall stress at buckling collapse relatively low, radial deflection due to membrane stresses would normally be low, therefore, end discontinuity stresses caused by the plug restraint would also be low. However, due to the variation of cylinder construction, some of these cylinders may be critical at the ends. The thick walled versions of the two highest buckling strength constructions, to be tested later in the program for compressive strength, are expected to present definite end shear problems.

An analysis is therefore being performed on all cylinder configurations to determine these stresses. This analysis is based on a technique for determining dome to cylinder discontinuity stresses in FRP motor cases developed at Douglas [6]. The cylinder ends are assumed to be completely restrained against radial deflections and rotations. This condition results in the highest moment and shear loads on the cylinder and conservatively represents the actual conditions.

Table 3 shows the maximum moment, maximum shear loading, "composite" or average stresses on the cylinder wall, maximum laminate and "local" stresses at significant locations through the cylinder wall, for constructions checked up to this time.

4.0 FABRICATION

4.1 Tooling

Special mandrels with auxiliary fixtures have been designed for the fabrication of the test cylinders. The design of these mandrels was based on the following objectives:

1. Sufficient length to fabricate both test cylinders for each configuration and a number of short rings for miscellaneous physical and mechanical tests at one time and with a minimum of variation.

2. Internal heating capacity to provide elevated temperature winding operations and cure of cylinders from the inside with a minimum of cost and elaborate equipment.

3. If possible without a costly and elaborate system, some provision for tensioning longitudinal filaments.

4. Dual usage of the same mandrel for both hoop and longitudinal constructions and helically wound constructions.

5. Ease of cylinder removal after cure.

Figure 6 illustrates the important features of the mandrel construction. Two (2) such mandrels will be built and will share a common commutator. Provision for two mandrels will allow winding operations to proceed continuously while a wound cylinder is being cured. It will also allow for simultaneous fabrication of hoop and longitudinal configurations and wet wound helical configurations in different machines. Due to the tightness of the revised schedule this may be necessary to complete all fabrication and testing by the end of the calendar year if any delays should arise.

Internal Chromalox C-512 cartridge heaters are supplied with power through the shaft mounted commutator. Temperature control is accomplished by the internally mounted Chromalox SA-501 thermostat. Heat paths are provided from internal heaters to outer mandrel shell by the closely spaced aluminum ribs and temperature variation between points on the surface is expected to be within a few degrees. The arrangement allows elevated temperature winding operations and cure from the inside to be accomplished using a single heat source and without external heating apparatus which might interfere with the technician's freedom of action. The arrangement would also lend itself to simple step curing procedures for thick-wall cylinders should such methods be advisable in the future.

The grooved end fittings are designed to provide tension on the longitudinal laminates via "overwrap" hoop windings into the grooves. This technique has been successfully applied in another program performed at Douglas. The fittings will also be used to aide in the removal of the cylinder from the mandrel. These fittings will be removed and replaced with "dummy" domes provided with cut-off slots when the mandrel is used for the wet winding of helically wound cylinders.

To reduce possible cylinder removal problems, the cylinder is constructed of a thick wall (1") aluminum tube and machined to provide a slightly tapered diameter from one end to the other. The taper is not expected to affect the buckling strength of the cylinders or interfere with the test procedure. A fine surface finish and the use of teflon parting compound will also aid in cylinder removal.

During the last quarter, the detail design of these mandrels was completed, drawings were released, materials were ordered and received and fabrication work was begun. The mandrels are scheduled for completion by late June.

4.2 <u>Test Cylinders</u>

All test cylinders except helically wound configurations will be fabricated from Douglas collimated pre-preg tapes. Figure 7 illustrates some of the tape and winding details.

Due to anticipated scheduling problems on the tape making machine during the next period, fabrication of tapes for this program has been started during this quarter using materials borrowed from stocks on hand. Tapes will be stored under refrigeration until needed. Fabrication of test cylinders will begin as soon as mandrels are available.

5.0 OUTLINE OF FUTURE WORK

A brief summary of work scheduled for the next quarter follows:

1. Theoretical Studies and Test Cylinder Design

   a. Further review of the assumptions made and models used to derive analytical expressions in the preliminary studies will be made. Comparison of theoretical values of elastic constants with continuing company-funded test programs and with cylinders fabricated in this period will be made.

   b. Detail design of cylindrical test specimens will be completed and fabrication drawings released.

   c. Buckling and disconinuity stress analysis for the test cylinders will be completed.

2. Tooling

   Fabrication of mandrels and auxiliary equipment will be completed and sample windings made to check out the mandrels.

3. Douglas Pre-preg Tape

   Fabrication of sufficient Douglas collimated pre-preg tape to complete the program will be completed. Tape will be stored under refrigeration until used.

4. Fabrication

   Fabrication of Group I test cylinders will be completed and Group II test cylinder initiated. Group I cylinders will be instrumented in preparation for hydrostatic test.

5. Testing

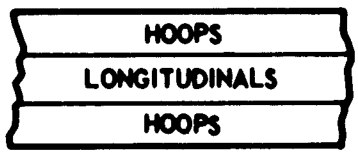
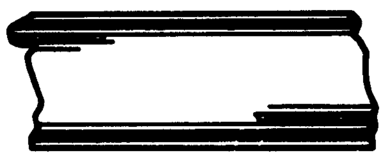
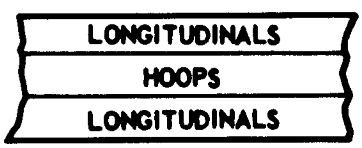
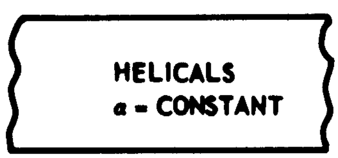
   Resin determination and miscellaneous physical property tests on ring samples cut from completed test cylinders will be performed. No hydrostatic testing of cylinders is scheduled for this period.

REFERENCES

1. Proposal to Determine the Optimum Construction of Reinforced Plastic Cylinders Subjected to High External Pressure, Douglas Report SM-42612, Dec. 1962.
2. Schneider, M. H., General Instability of Orthotropic Cylinders Subjected to Axial Compression and External Pressure, Douglas Co. Report SM-41986, June 1962.
3. Hofeditz, J. T., and Schneider, M. H., Buckling of Fiberglas Cylinders Under External Pressure, Douglas Co. Report SM-42074, August, 1962.
4. Greszczuk, L. B., Membrane Analysis Methods for Composite Structures, Douglas Co. Report SM-41543.
5. Stein, M. and Mayers, J., A Small Deflection Theory for Curved Sandwich Plates, NACA Report 1008, 1951.
6. Wills, C., Discontinuity Stresses at a Dome-Cylinder Joint in Fiberglas Motor Cases, unpublished Douglas Co. report.

TABLE I
FABRICATION AND TEST REQUIREMENTS

I.D. = 6" L = 12" t = .150 (EXCEPT AS NOTED BY*)

TYPE OF CONSTRUCTION	SYMBOL	K (% HOOPS)	NO. TO BE TESTED
OPTIMUM DESIGN	A	≈ .95	2
	B	.667	3
	C	.333	2
	D	≈ .05	2
	M*	.667	1
FULLY DISPERSED	E	.667	3
	F	.333	2
	N*	.667	1
HIGH AXIAL STRENGTH	G	.667	2
	H	.333	2
HELICALLY WOUND		α	
	J	≈ 85°	2
	K	60°	2
	L	30°	2
		TOTAL	26

*COMPRESSION TESTS – t = ½

TABLE 2

RESULTS OF BUCKLING ANALYSIS

CONFIGURATION	B_x (lb/in x 106)	$\frac{D_y}{(1-\mu_x \mu_y)}$ (lb/in^2)	Z_y	p_{cr} (psi)	$\frac{p_{cr}}{p_{cr_{MAX}}}$	$\sigma_y = \frac{p_{cr} r}{t}$(1) (psi)
A	.4415	1,970	536	740	.93	15,200
B	.6172	1,940	638	795	1.00	16,300
C	.8235	1,620	806	754	.95	15,500
D	1.0	955	1,160	530	.67	10,900
E	.617	1,590	706	683	.86	14,000
F	.824	1,178	950	588	.74	12,100
G	.617	1,140	843	535	.67	11,000
H	.8235	833	1,130	455	.57	9,330
J						
K	.346	1,320	582	517	.65	10,600
L	.526	875	855	422	.53	8,650

(1) Average composite hoop stress at buckling.

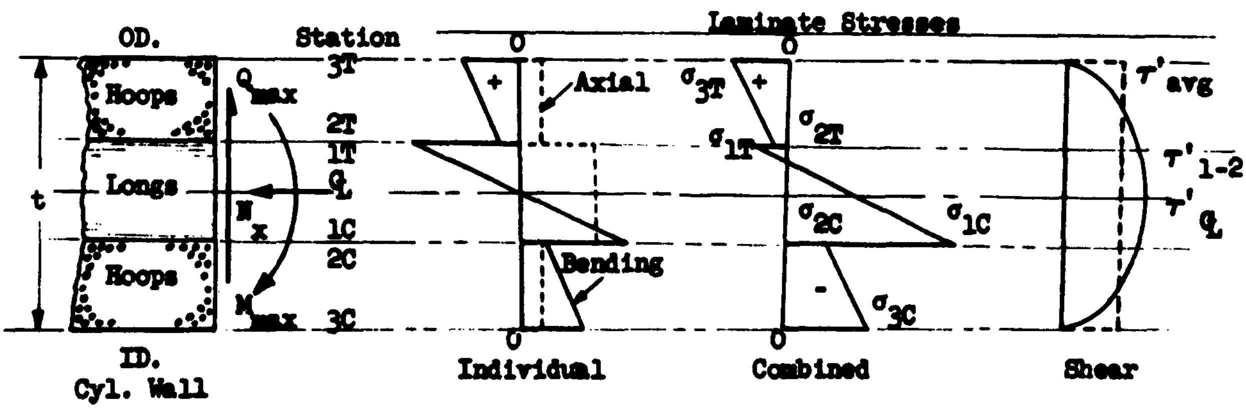
TABLE 3

RESULTS OF DISCONTINUITY STRESS ANALYSIS

CONSTRUCTION	HOOP-LONG-HOOP				DISPERSED	
CONFIGURATION	A	B	C	D	E	F
K, % Hoops	95	67	33	5	67	33
$\sigma'_x = \frac{P_{cr}r}{2t}$ (psi)(1)(2)	-7,590	-8,150	-7,730	-5,430	-7,000	-6,060
M_{max} (in/lb/in)	54.6	68	90	97.1	76.6	84
σ'_{outer} (psi)	2,010	9,950	16,270	20,370	13,400	16,340
σ_{3T} (psi)	7,240	11,370	12,050	8,810	8,540	8,180
σ_{2T} (psi)	-6,355	110	7,050	8,210	21,400	20,540
σ_{1T} (psi)	-15,910	400	17,350	20,600		
σ_{1C} (psi)	-19,490	-27,600	-36,650	-31,600		
σ_{2C} (psi)	-7,765	-10,920	-14,750	-12,590	-44,800	-35,660
σ_{3C} (psi)	-21,360	-22,230	-19,750	-13,190	-17,860	-14,220
σ'_{inner} (psi)	-22,190	-26,250	-31,170	-31,230	-27,400	-28,460
Q_{max} (lb/in)	266	311	350	308	238	300
τ'_{avg} (psi)	1,770	2,680	2,330	2,050	2,190	2,000
τ'_{1-2} (psi)	2,660	2,940	795	93		
$\tau'_{\text{℄}}$ (psi)	3,620	3,840	2,400	2,210	3,280	3,000

(1) Tension stresses shown positive, compression stresses shown negative.

(2) Primes refer to stresses computed using isotropic equations, e.g., $\sigma = \frac{pr}{2t} +$



RELATIVE BUCKLING STRENGTH OF EQUAL WEIGHT CYLINDERS UNDER EXTERNAL PRESSURE

$\frac{E_r}{E_\theta} = .050$

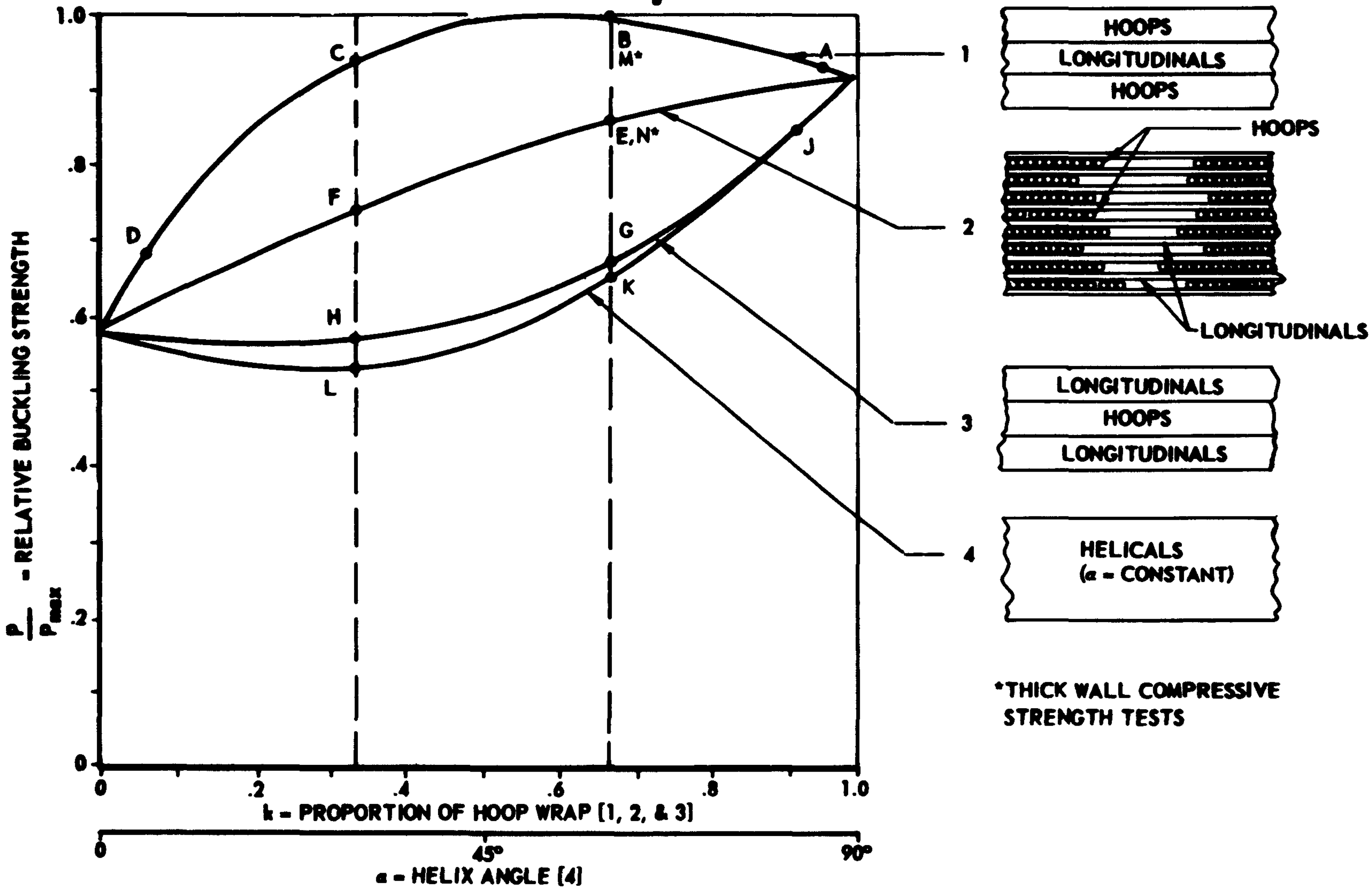


FIGURE 1

Optimum Construction of Reinforced Plastic Cylinders to Resist High External Pressure - NObs-88425

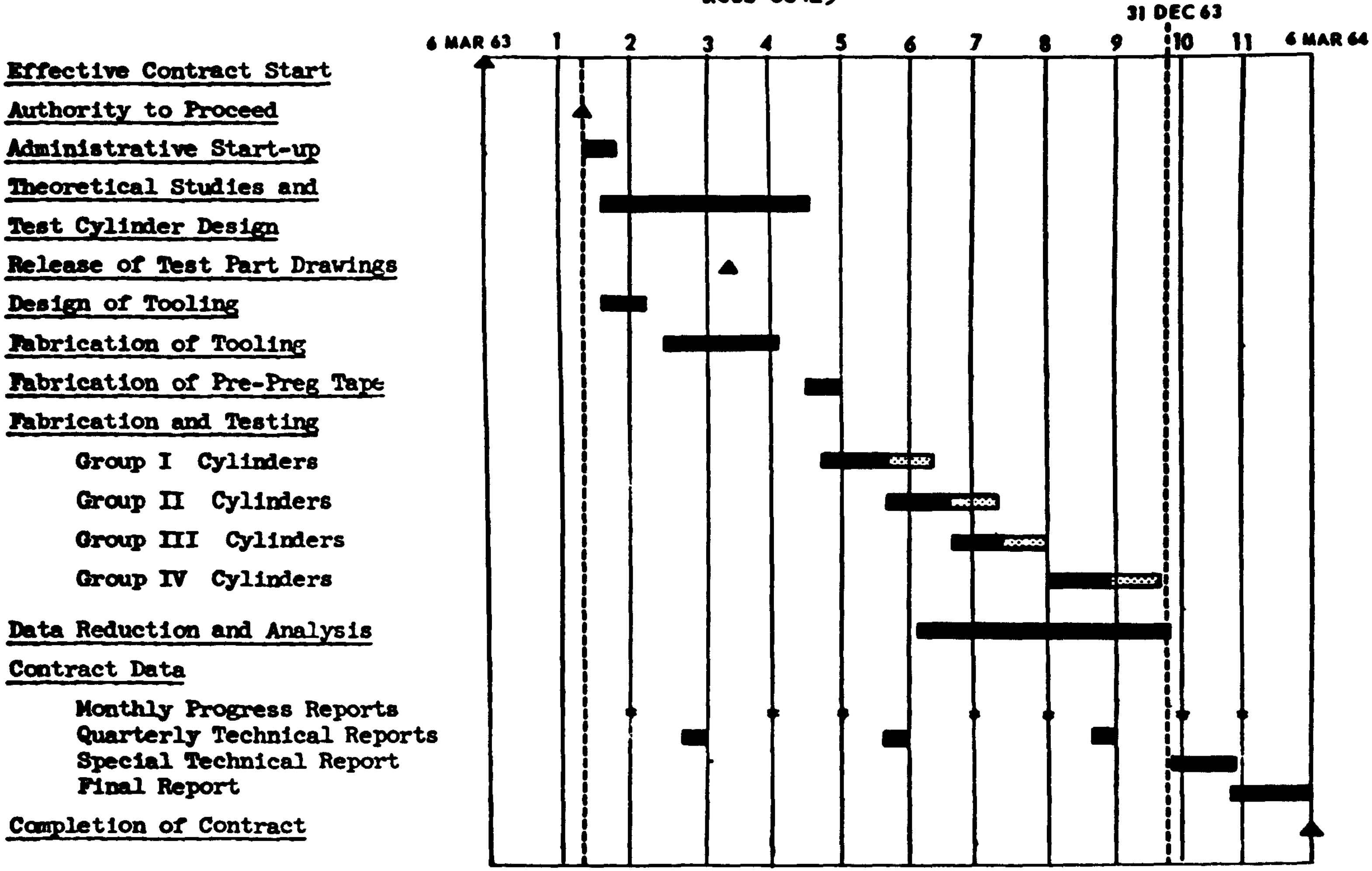


FIGURE 2 PROGRAM SCHEDULE

Ref. SM-42612

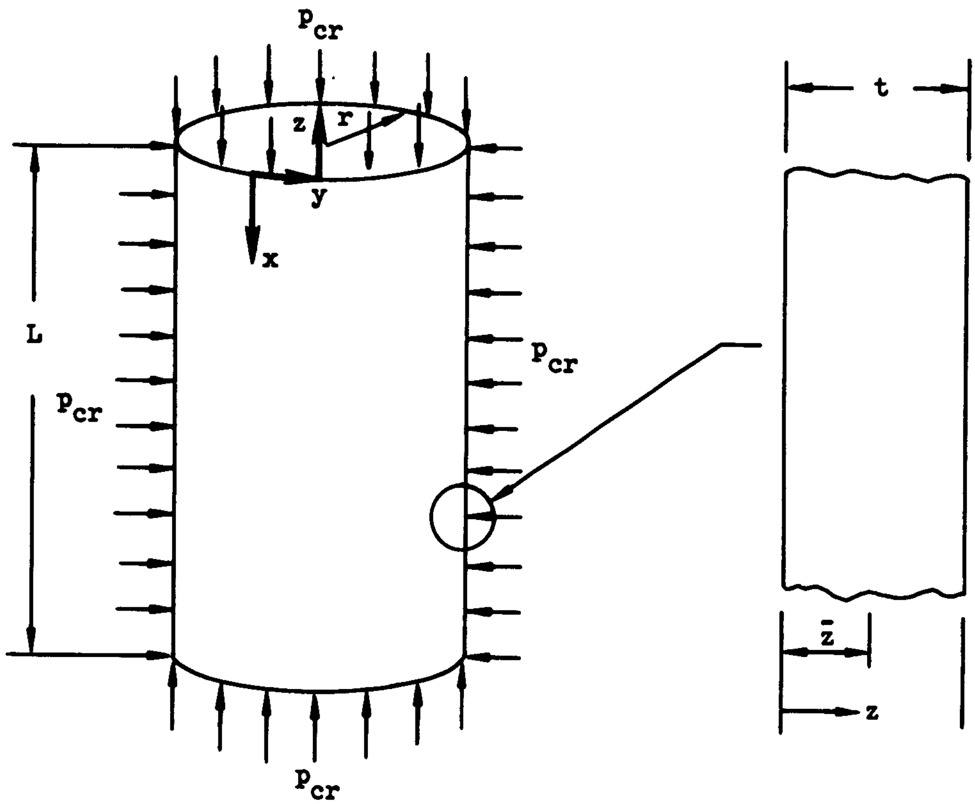
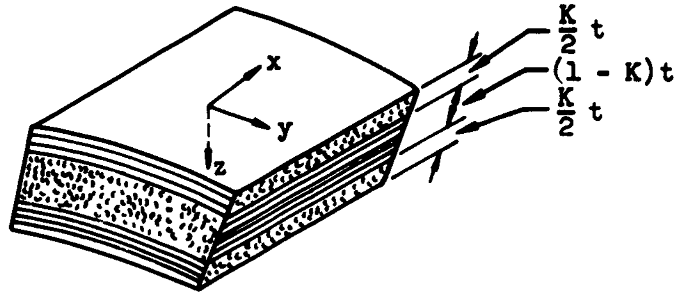


FIGURE 3. CYLINDER UNDER EXTERNAL HYDROSTATIC PRESSURE.

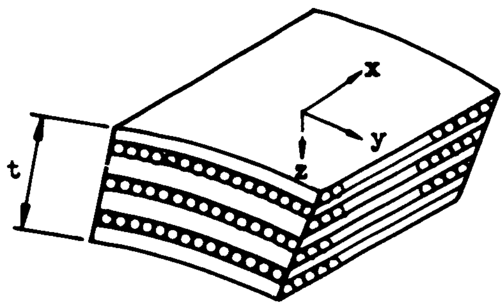
1 - OPTIMUM DESIGN - (HOOP LAMINATES WITH LONGITUDINAL INCENTER)



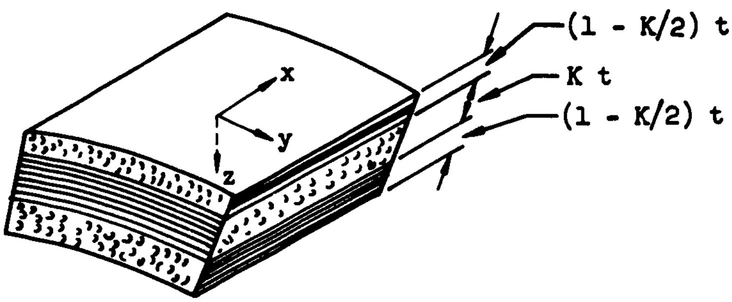
2 - DISPERSED CONSTRUCTION

Kt = TOTAL THICKNESS OF HOOPS

(1-K)t = TOTAL THICKNESS OF LONGITUDINALS



3 - HIGH AXIAL STRENGTH (LONGITUDINALS WITH HOOPS IN CENTER)



4 - CONSTANT HELIX ANGLE

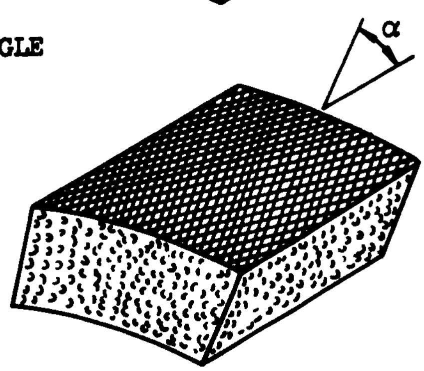
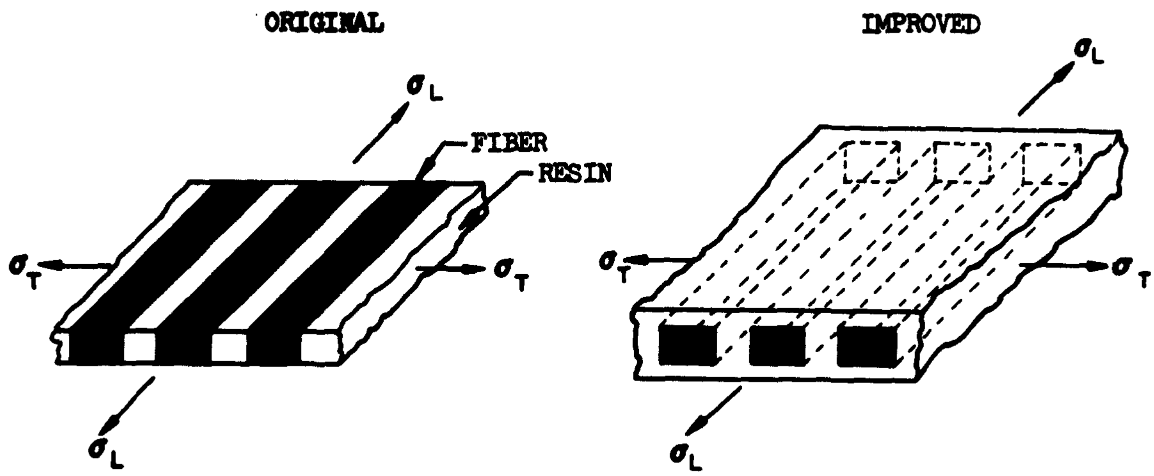


FIG. 4 TYPES OF LAMINATE CONSTRUCTIONS



ELASTIC CONSTANTS

$$E_L = \lambda E_g + (1-\lambda) E_r \qquad\qquad E_L = \lambda E_g + (1-\lambda) E_r$$

$$E_r = \frac{E_r}{(1-\lambda) + \frac{E_r}{E_g}\lambda} \qquad\qquad E_T = \frac{\frac{E_r}{E_g}\left[(1-\sqrt{\lambda})\frac{E_r}{E_g} + \sqrt{\lambda}\right]}{\frac{E_r}{E_g}\sqrt{\lambda} + (1-2\mu_r^2)(1-\sqrt{\lambda})\left[(1-\sqrt{\lambda})\frac{E_r}{E_g} + \sqrt{\lambda}\right]}$$

$$G_{LT} = \frac{G_r G_g}{G_g (1-\lambda) + G_r \lambda} \qquad\qquad G_{LT} = \frac{G_r G_g\left[\sqrt{\lambda} + \frac{G_r}{G_g}(1-\sqrt{\lambda})\right]}{G_g (1-\sqrt{\lambda})\sqrt{\lambda} + G_r\left[(1-\sqrt{\lambda})^2 + \sqrt{\lambda}\right]}$$

Figure 5 - Unidirection Laminate Models

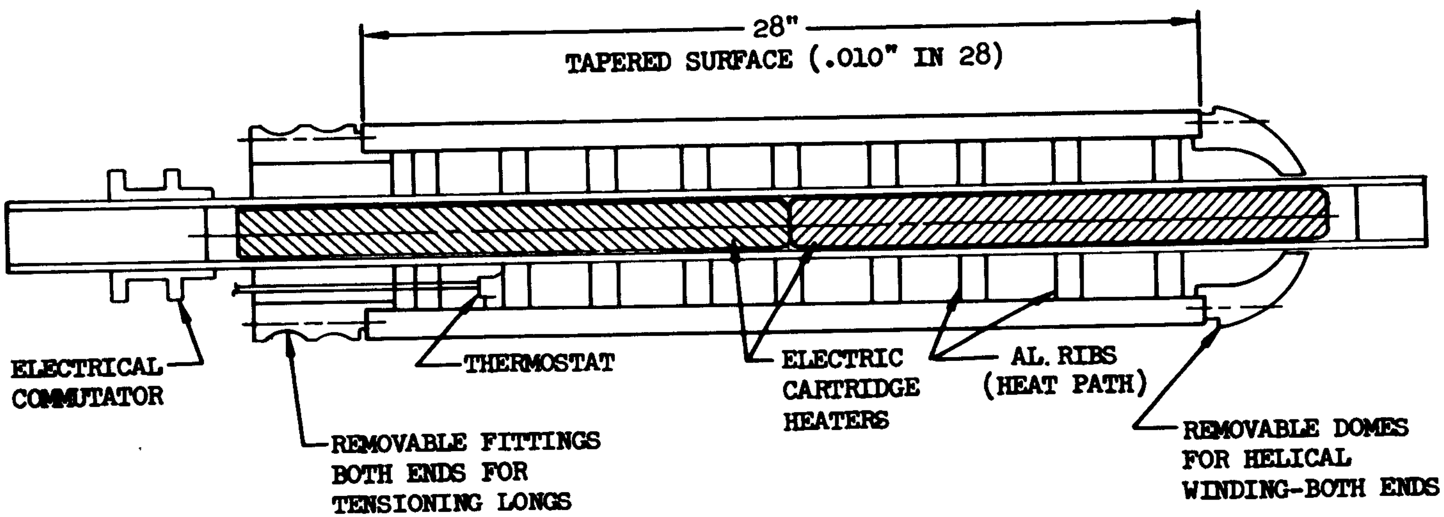
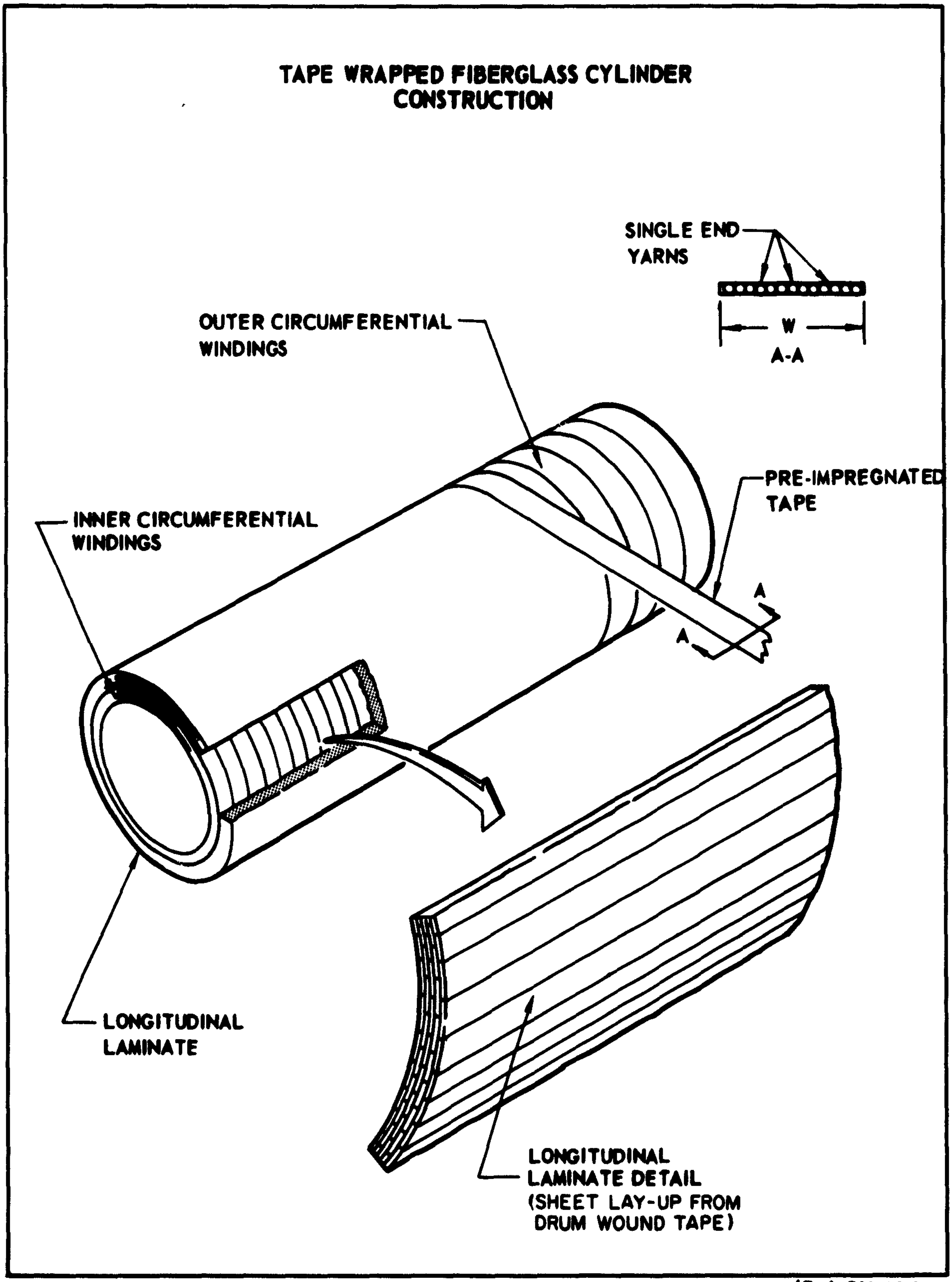


FIG 6 INTERNALLY HEATED ALUMINUM MANDREL



(Ref SM 42612)

FIGURE 7

APPENDIX A

SOLUTION OF DIFFERENTIAL EQUATION FOR BUCKLING OF ORTHOTROPIC CYLINDERS UNDER EXTERNAL PRESSURE

The buckling equations for an orthotropic monocoque cylinder are derived below. Also a stiffened cylindrical shell under external pressure may be considered an orthotropic shell provided buckling occurs across several stiffeners. The method of solution is then similar to that given in ref. |2| for axial compression. For external pressure loading the general equations of equilibrium developed in reference |5| reduce to

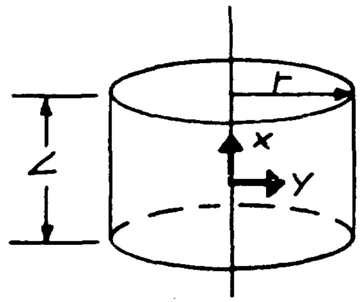


Figure 8. Cylinder Geometry and Coordinate System

$$L_D\, w + \frac{G_{xy}}{r^2} L_E^{-1} \frac{\partial^4 w}{\partial x^4} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \qquad \text{(A1)}$$

where L_D and L_E are linear differential operators defined by

$$L_D = \frac{D_x}{1-\mu_x \mu_y} \frac{\partial^4}{\partial x^4} + \left(\frac{\mu_y D_x}{1-\mu_x \mu_y} + 2 D_{xy} + \frac{\mu_x D_y}{1-\mu_x \mu_y}\right) \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{D_y}{1-\mu_x \mu_y} \frac{\partial^4}{\partial y^4}$$

$$L_E = \frac{G_{xy}}{B_y} \frac{\partial^4}{\partial x^4} + \left(1 - \mu'_x \frac{G_{xy}}{B_x} - \mu'_y \frac{G_{xy}}{B_y}\right) \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{G_{xy}}{B_x} \frac{\partial^4}{\partial y^4}$$

and L_E^{-1} is the inverse operator defined by

$$L_E^{-1} (L_E\, w) = L_E (L_E^{-1}\, w) = w$$

EXTERNAL RADIAL PRESSURE

For buckling under uniform external radial pressure equation (A1) becomes, since $N_x = 0$,

$$L_D w + \frac{G_{xy}}{r^2} L_E^{-1} \frac{\partial^4 w}{\partial x^4} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \qquad \text{(A2)}$$

The boundry conditions will be satisfied if the displacement is taken as

$$w = w_o \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$$

For simply supported ends.

$\frac{\partial w}{\partial x} = w_o\left(\frac{m\pi}{L}\right) \cos \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$	$\frac{\partial w}{\partial y} = w_o\left(\frac{n\pi}{b}\right) \sin \frac{m\pi x}{L} \cos \frac{n\pi y}{b}$
$\frac{\partial^2 w}{\partial x^2} = -w_o\left(\frac{m\pi}{L}\right)^2 \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$	$\frac{\partial^2 w}{\partial y^2} = -w_o\left(\frac{n\pi}{b}\right)^2 \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$
$\frac{\partial^3 w}{\partial x^3} = -w_o\left(\frac{m\pi}{L}\right)^3 \cos \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$	$\frac{\partial^3 w}{\partial y^3} = -w_o\left(\frac{n\pi}{b}\right)^3 \sin \frac{m\pi x}{L} \cos \frac{n\pi y}{b}$
$\frac{\partial^4 w}{\partial x^4} = w_o\left(\frac{m\pi}{L}\right)^4 \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$	$\frac{\partial^4 w}{\partial y^4} = w_o\left(\frac{n\pi}{b}\right)^4 \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$

$$L_D = \left[\frac{D_x}{1-\mu_x\mu_y}\left(\frac{m\pi}{L}\right)^4 + \left(\frac{\mu_y D_x}{1-\mu_x\mu_y} + 2D_{xy} + \frac{\mu_x D_y}{1-\mu_x\mu_y}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{b}\right)^2 + \frac{D_y}{1-\mu_x\mu_y}\left(\frac{n\pi}{b}\right)^4\right] w_o \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$$

$$L_E = \left[\frac{G_{xy}}{B_y}\left(\frac{m\pi}{L}\right)^4 + \left(1-\mu_x' \frac{G_{xy}}{B_x} - \mu_y' \frac{G_{xy}}{B_y}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{b}\right)^2 + \frac{G_{xy}}{B_x}\left(\frac{n\pi}{b}\right)^4\right] w_o \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$$

$$L_E^{-1} \frac{\partial^4 w}{\partial x^4} = \frac{\left(\frac{m\pi}{L}\right)^4}{\left[\frac{G_{xy}}{B_y}\left(\frac{m\pi}{L}\right)^4 + \left(1-\mu_x' \frac{G_{xy}}{B_x} - \mu_y' \frac{G_{xy}}{B_y}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{b}\right)^2 + \frac{G_{xy}}{B_x}\left(\frac{n\pi}{b}\right)^4\right]} w_o \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$$

Equation (A1) becomes

$$\frac{D_x}{1-\mu_x\mu_y}\left(\frac{m\pi}{L}\right)^4 + \left(\frac{\mu_y D_x}{1-\mu_x\mu_y} + 2D_{xy} + \frac{\mu_x D_y}{1-\mu_x\mu_y}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{b}\right)^2 + \frac{D_y}{1-\mu_x\mu_y}\left(\frac{n\pi}{b}\right)^4 + \frac{G_{xy}}{r^2}\frac{\left(\frac{m\pi}{L}\right)^4}{\frac{G_{xy}}{B_y}\left(\frac{m\pi}{L}\right)^4 + \left(1-\mu_x' \frac{G_{xy}}{B_x} - \mu_y' \frac{G_{xy}}{B_y}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{b}\right)^2 + \frac{G_{xy}}{B_x}\left(\frac{n\pi}{b}\right)^4} - N_y\left(\frac{n\pi}{b}\right)^2 = 0 \qquad \text{(A3)}$$

$$N_y = \left(\frac{n\pi}{L}\right)^2\left[\frac{D_x}{1-\mu_x\mu_y}\frac{1}{\left(\frac{n}{m}\right)^2\left(\frac{L}{b}\right)^2} + \left(\frac{\mu_y D_x}{1-\mu_x\mu_y} + 2D_{xy} + \frac{\mu_x D_y}{1-\mu_x\mu_y}\right) + \frac{D_y}{1-\mu_x\mu_y}\left(\frac{n}{m}\right)^2\left(\frac{L}{b}\right)^2\right] + \frac{1}{r^2}\frac{1}{\left(\frac{n\pi}{L}\right)^2\left[\frac{1}{B_x}\left(\frac{n}{m}\right)^2\left(\frac{L}{b}\right)^2 + \left(\frac{\mu_x'}{B_y} + \frac{1}{G_{xy}} + \frac{\mu_y'}{B_x}\right)\left(\frac{n}{m}\right)^4\left(\frac{L}{b}\right)^4 + \frac{1}{B_y}\left(\frac{n}{m}\right)^6\left(\frac{L}{b}\right)^6\right]} \qquad \text{(A4)}$$

Letting $\lambda' = \frac{L}{m}$ = half-wave length of buckle in longitudinal direction and multiplying by $\frac{L^2 (1-\mu_x \mu_y)}{\pi^2 D_y}$

$$\frac{N_y L^2 (1-\mu_x \mu_y)}{\pi^2 D_y} = m^2 \left[\frac{D_x}{D_y} \frac{1}{(\lambda' \frac{n}{b})^2} + \left(\mu_y \frac{D_x}{D_y} + \frac{2D_{xy} (1-\mu_x \mu_y)}{D_y} + \mu_x\right) + (\lambda' \frac{n}{b})^2 \right]$$

$$+ \frac{L^4 (1-\mu_x \mu_y) B_x}{r^2 D_y} \frac{1}{\pi^4 m^2} \frac{1}{\left[\frac{B_x}{B_y} (\lambda' \frac{n}{b})^2 - \left(\mu'_y \frac{B_x}{B_y} - \frac{B_x}{G_{xy}} + \mu'_x\right) (\lambda' \frac{n}{b})^4 + (\lambda' \frac{n}{b})^6 \right]} \quad \text{(A5)}$$

Define: $\frac{L^4 (1-\mu_x \mu_y)}{r^2} \frac{B_x}{D_y} = Z_y^2$ and $\frac{N_y L^2 (1-\mu_x \mu_y)}{\pi^2 D_y} = K_y$

$$\therefore Z_y = \frac{L^2}{r} \sqrt{1-\mu_x \mu_y} \sqrt{\frac{B_x}{D_y}}$$

Equation (A5) can now be written in terms of Z_y. Using the reciprocal relationship $\mu'_x B_y = \mu'_y B_x$ and $\mu_x D_y = \mu_y D_x$

$$K_y = m^2 \left[\frac{D_x}{D_y} \frac{1}{(\lambda' \frac{n}{b})^2} + 2 \left(\mu_x + \frac{D_{xy}}{D_y} (1-\mu_x \mu_y)\right) + (\lambda' \frac{n}{b})^2 \right] + \frac{Z_y^2}{\pi^4} \frac{1}{m^2} \frac{1}{\left[\frac{B_x}{B_y} (\lambda' \frac{n}{b})^2 - 2 \left(\mu'_x - \frac{B_x}{2G_{xy}}\right) (\lambda' \frac{n}{b})^4 + (\lambda' \frac{n}{b})^6 \right]} \quad \text{(A6)}$$

Buckling of Long Orthotropic Cylinders.

The critical pressure can be determined by taking the ratio of axial half-wave length to circumferential half-wave length as much greater than unity and minimizing with respect to $(\lambda' \frac{n}{b})^2$. From equation (A6) if $(\lambda' \frac{n}{b})^2 >> 1$ we have

$$K_y = m^2 (\lambda' \frac{n}{b})^2 + \frac{Z_y^2}{\pi^4} \frac{1}{m^2 (\lambda' \frac{n}{b})^6} \tag{A7}$$

Differentiating with respect to $(\lambda' \frac{n}{b})^2$ gives

$$\frac{\partial K_y}{\partial (\lambda' \frac{n}{b})^2} = 2\, m^2 (\lambda' \frac{n}{b}) - 6 \frac{Z_y^2}{\pi^4} \frac{1}{m^2 (\lambda' \frac{n}{b})^7} \tag{A8}$$

Since the second derivative of K_y with respect to $(\lambda' \frac{n}{b})^2$ is obviously positive, setting equation (A8) equal to zero gives a minimum of K_y for given values m^2. Setting equation (A8) equal to zero gives

$$(\lambda' \frac{n}{b})^2 = \frac{\sqrt[4]{3}}{\pi} \frac{\sqrt{Z_y}}{m}$$

Substituting the above value of $(\lambda' \frac{n}{b})^2$ into equation (A7) gives

$$K_y = \frac{4 \sqrt[4]{3}}{3} \frac{1}{\pi}\, m \sqrt{Z_y}$$

from which $m = 1$ gives the minimum value of K_y.

$$\boxed{K_y = 0.558 \sqrt{Z_y}} \tag{A9}$$

The critical load is found from

$$K_y = \frac{N_y L^2 (1-\mu_x \mu_y)}{\pi^2 D_y} = 0.558 \, Z_y^{1/2}$$

$$N_y = \pi^2 (0.558) \frac{D_y}{L^2(1-\mu_x \mu_y)} Z_y^{1/2} = \frac{5.51}{L^2} \frac{D_y}{(1-\mu_x \mu_y)} \left[\frac{L^2}{r} \sqrt{1-\mu_x \mu_y} \left(\frac{B_x}{D_y}\right)^{1/2} \right]^{1/2}$$

$$\boxed{N_y = p_{cr} r = \frac{5.51}{L \, r^{1/2}} \left(\frac{D_y}{1-\mu_x \mu_y}\right)^{3/4} B_x^{1/4}} \qquad \text{(A10)}$$

EXTERNAL HYDROSTATIC PRESSURE

For buckling under external hydrostatic pressure, equation (A1) becomes

$$L_D w + \frac{G_{xy}}{r^2} L_E^{-1} \frac{\partial^4 w}{\partial x^4} + \frac{1}{2} N_y \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \qquad \text{(A11)}$$

Using the displacement function

$$w = w_o \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{b}$$

Equation (A11) becomes (ends simply supported) -

$$N_y\left[\frac{1}{2}\left(\frac{m\pi}{L}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]=\frac{D_x}{1-\mu_x\mu_y}\left(\frac{m\pi}{L}\right)^4+\left(\frac{\mu_y D_x}{1-\mu_x\mu_y}+2D_{xy}+\frac{\mu_x D_y}{1-\mu_x\mu_y}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{b}\right)^2+\frac{D_y}{1-\mu_x\mu_y}\left(\frac{n\pi}{b}\right)^4+\frac{B_x}{r^2}\frac{\left(\frac{m\pi}{L}\right)^4}{\frac{B_x}{B_y}\left(\frac{m\pi}{L}\right)^4+\left(1-\mu_y'\frac{B_x}{B_y}-\mu_x'\frac{B_x}{B_y}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n\pi}{b}\right)^2+\frac{B_x}{B_y}\left(\frac{n\pi}{b}\right)^4} \quad \text{(A12)}$$

Dividing by $\left(\frac{n\pi}{b}\right)^2$ and noting $\lambda'=\frac{L}{m}$

$$N_y\left[\frac{1}{2}\frac{1}{\left(\lambda'\frac{n}{b}\right)^2}+1\right]=\left(\frac{m\pi}{L}\right)^2\left[\frac{D_x}{1-\mu_x\mu_y}\frac{1}{\left(\lambda'\frac{n}{b}\right)^2}+\left(\frac{\mu_y D_x}{1-\mu_x\mu_y}+2D_{xy}+\frac{\mu_x D_y}{1-\mu_x\mu_y}\right)+\frac{D_y}{1-\mu_x\mu_y}\left(\lambda'\frac{n}{b}\right)^2\right]+\frac{1}{r^2}\frac{B_x}{\left(\frac{m\pi}{L}\right)^2\left[\frac{B_x}{B_y}\left(\lambda'\frac{n}{b}\right)^{-2}-\left(\mu_y'\frac{B_x}{B_y}-\frac{B_x}{G_{xy}}-\mu_x'\right)\left(\lambda'\frac{n}{b}\right)^2-\left(\lambda'\frac{n}{b}\right)^4\right]}$$

$$N_y=\left(\frac{m\pi}{L}\right)^2\left[\frac{D_x}{1-\mu_x\mu_y}\frac{1}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}+\left(\frac{\mu_y D_x}{1-\mu_x\mu_y}+2D_{xy}+\frac{\mu_x D_y}{1-\mu_x\mu_y}\right)\frac{\left(\lambda'\frac{n}{b}\right)^2}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}+\frac{D_y}{1-\mu_x\mu_y}\frac{\left(\lambda'\frac{n}{b}\right)^4}{\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2}\right]+\frac{B_x}{r^2\left(\frac{m\pi}{L}\right)^2\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)\left[\frac{B_x}{B_y}-\mu_y'\frac{B_x}{B_y}-\frac{B_x}{G_{xy}}-\mu_x'\left(\lambda'\frac{n}{b}\right)^2+\left(\lambda'\frac{n}{b}\right)^4\right]} \quad \text{(A13)}$$

Multiplying by $\frac{L^2(1-\mu_x\mu_y)}{\pi^2 D_y}$ and using the reciprocal relationship $\mu_x' B_y=\mu_y' B_x$ and $\mu_x D_y=\mu_y D_x$

$$\frac{N_y L^2(1-\mu_x\mu_y)}{\pi^2 D_y}=m^2\left[\frac{D_x}{D_y}\frac{1}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}+2\left(\mu_x+\frac{D_{xy}(1-\mu_x\mu_y)}{D_y}\right)\frac{\left(\lambda'\frac{n}{b}\right)^2}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}+\frac{\left(\lambda'\frac{n}{b}\right)^4}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}\right]+\frac{L^4(1-\mu_x\mu_y)B_x}{r^2 D_y}\frac{1}{m^2\pi^4}\frac{1}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)\left[\frac{B_x}{B_y}-2\left(\mu_y'-\frac{B_x}{G_{xy}}\right)\left(\lambda'\frac{n}{b}\right)^2+\left(\lambda'\frac{n}{b}\right)^4\right]} \quad \text{(A14)}$$

Define: $\frac{L^4(1-\mu_x\mu_y)B_x}{r^2 D_y}=Z_y^2$ and $\frac{N_y L^2(1-\mu_x\mu_y)}{\pi^2 D_y}=K_y$

$$\therefore Z_y=\frac{L^2}{r}\sqrt{1-\mu_x\mu_y}\sqrt{\frac{B_x}{D_y}}$$

Equation (A14) becomes

$$K_y=m^2\left[\frac{D_x}{D_y}\frac{1}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}+2\left(\mu_x+\frac{D_{xy}(1-\mu_x\mu_y)}{D_y}\right)\frac{\left(\lambda'\frac{n}{b}\right)^2}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}+\frac{\left(\lambda'\frac{n}{b}\right)^4}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)}\right]+\frac{Z_y^2}{\pi^4}\frac{1}{m^2}\frac{1}{\left(\frac{1}{2}+\left(\lambda'\frac{n}{b}\right)^2\right)\left[\frac{B_x}{B_y}-2\left(\mu_y'-\frac{B_x}{G_{xy}}\right)\left(\lambda'\frac{n}{b}\right)^2+\left(\lambda'\frac{n}{b}\right)^4\right]} \quad \text{(A15)}$$

Buckling of Long Orthotropic Cylinders

Equation (A15) can be solved for the long cylinder under hydrostatic pressure by taking the number of half-waves in the axial direction as unity and minimizing with respect to $\left(\lambda'\frac{n}{b}\right)^2$. If the ratio of half-wave length in the axial direction to half-wave length in the circumferential direction is much greater than unity, then the solution for a long cylinder under hydrostatic pressure reduces to the solution for a long cylinder under radial pressure and equations (A9) and (A10) will apply.

APPENDIX B

MODULI OF ELASTICITY AND POISSON'S RATIOS

The elastic moduli, shear modulus, and Poisson's ratios for a laminate (Figure 9) in terms of the material properties of the glass and resin and glass content, λ, are given in Reference [4] as:

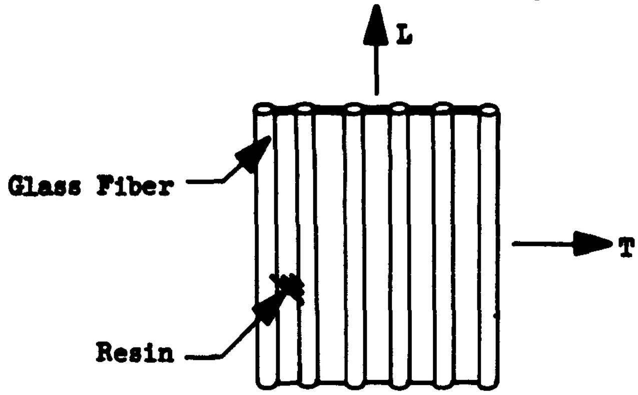


Figure 9. Single Laminate.

$$\frac{E_L}{E_g} = \lambda + \frac{E_r}{E_g}(1 - \lambda) \tag{B1}$$

$$\frac{E_T}{E_g} = \frac{\frac{E_r}{E_g}\left[(1 - \sqrt{\lambda})\frac{E_r}{E_g} + \sqrt{\lambda}\right]}{\sqrt{\lambda}\frac{E_r}{E_g} + (1 - \sqrt{\lambda})\left[(1 - \sqrt{\lambda})\frac{E_r}{E_g} + \sqrt{\lambda}\right](1 - 2\mu_r)^2} \tag{B2*}$$

$$G_{LT} = \frac{G_r G_g\left[\sqrt{\lambda} + \frac{G_r}{G_g}(1-\sqrt{\lambda})\right]}{G_g(1-\sqrt{\lambda})\sqrt{\lambda} + G_r\left[(1-\sqrt{\lambda})^2 + \sqrt{\lambda}\right]} \quad *$$

where

$$G_r = \frac{E_r}{2(1+\mu_r)} \qquad \text{and } G_g = \frac{E_g}{2(1+\mu_g)}$$

* Revised equation

$$\frac{G_{LT}}{E_g} = \frac{\frac{E_r}{2E_g}\left[\sqrt{\lambda} + \frac{E_r}{E_g}\left(\frac{1+\mu_g}{1+\mu_r}\right)(1-\sqrt{\lambda})\right]}{\sqrt{\lambda}\,(1-\sqrt{\lambda})(1+\mu_r) + \frac{E_r}{E_g}(1+\mu_g)\left[(1-\sqrt{\lambda})^2 + \sqrt{\lambda}\right]} \qquad \text{(B3)*}$$

$$\mu_{LT} = \mu_g\,\lambda + \mu_r\,(1-\lambda) \qquad \text{(B4)}$$

$$\mu_{TL} = \frac{\frac{E_T}{E_g}\mu_{LT}}{\frac{E_L}{E_g}} = \frac{\left[\mu_g\,\lambda + \mu_r\,(1-\lambda)\,\frac{E_r}{E_g}\right]}{\left[1-\lambda+\frac{E_r}{E_g}\lambda\right]\left[\lambda+\frac{E_r}{E_g}(1-\lambda)\right]} \qquad \text{(B5)}$$

When the fibers are oriented in the hoop direction, Equations (B1) through (B5) take on the following notation

$$\frac{E_L}{E_g} = \frac{E_{Y_H}}{E_g} \qquad\qquad \mu_{LT} = \mu_{Y_H}$$

$$G_{LT} = G_H \qquad \text{(B6)}$$

$$\frac{E_T}{E_g} = \frac{E_{X_H}}{E_g} \qquad\qquad \mu_{TL} = \mu_{X_H}$$

For fibers oriented in the longitudinal direction

$$\frac{E_L}{E_g} = \frac{E_{X_L}}{E_g} \qquad\qquad \mu_{LT} = \mu_{Y_L} \qquad \text{(B7)}$$

$$G_{LT} = G_L \tag{B7 (cont.)}$$

$$\frac{E_T}{E_g} = \frac{E_{Y_L}}{E_g} \qquad \mu_{T_L} = \mu_{X_L}$$

The elastic moduli, shear modulus, and Poisson's ratios for any balanced laminate (Figure 10) made from two single laminates are given by (Reference 3)

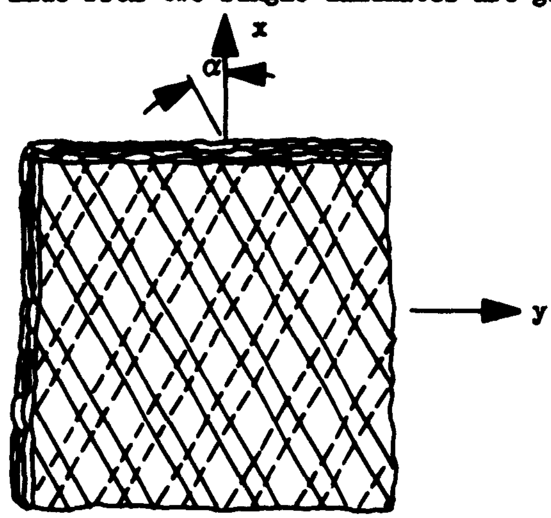


Figure 10. Balanced Laminate.

$$\frac{E_{X_{HW}}}{E_L} = \frac{1}{\cos^4 \alpha + \frac{E_L}{E_T} \sin^4 \alpha + \left(\frac{E_L}{G_{LT}} - 2\mu_{LT}\right) \sin^2 \alpha \cos^2 \alpha} \tag{B8}$$

$$\frac{E_{Y_{HW}}}{E_L} = \frac{1}{\sin^4 \alpha + \frac{E_L}{E_T} \cos^4 \alpha + \left(\frac{E_L}{G_{LT}} - 2\mu_{LT}\right) \sin^2 \alpha \cos^2 \alpha} \tag{B9}$$

$$\frac{G'_{XY_{HW}}}{E_L} = \frac{1}{1 + 2\mu_{LT} + \frac{E_L}{E_T} - \left(1 + 2\mu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{G_{LT}}\right)(\cos^2 \alpha - \sin^2 \alpha)^2} \tag{B10}$$

$$\mu_{X_{HW}} = \frac{E_{X_{HW}}}{E_L}\left[\mu_{LT} - \left(1 + 2\mu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{G_{LT}}\right) \sin^2 \alpha \cos^2 \alpha \right] \tag{B11}$$

$$\mu_{Y_{HW}} = \frac{E_{Y_{HW}}}{E_L}\left[\mu_{LT} - \left(1 + 2\mu_{LT} + \frac{E_L}{E_T} - \frac{E_L}{G_{LT}}\right) \sin^2 \alpha \cos^2 \alpha \right] \tag{B12}$$

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by H. R. Jacobson

Douglas Company Report No. SM-44057, 6 June 1963, pp. incl. illus., tables, appendices (Contract NObs-88425).

The purpose of this program is to develop and prove out a theoretical method for accurately predicting the critical buckling strength of externally pressurized cylinders made from orthotropic materials and to find the optimum construction of filament reinforced plastic (FRP) cylinders under such loading.

A description of the program and its significance to the overall BuShips Deep Submergency program and a summary of the work accomplished to date and that to be performed in the next period is given. Theoretical analytical expressions for predicting elasticity buckling collapse and elastic constants of FRP cylinders are presented and discussed. Results of initial buckling and discontinuity stress analysis of proposed test cylinders are presented.

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MISSILE & SPACE SYSTEMS DIVISION
DOUGLAS AIRCRAFT COMPANY, INC.
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14 June 1963

A-13P-494

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Very truly yours,

MISSILE & SPACE SYSTEMS DIVISION
Douglas Aircraft Company, Inc.

W. H. Scott

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